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PENMAN'S WIND FUNCTION FOR A TROPICAL HUMID CLIMATE[1]

A FUNÇÃO DO VENTO DE PENMAN PARA CLIMA TROPICAL ÚMIDO

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SUMMARY

Penman's wind function [f(u) = **m** (**a** + **b** u)] was calibrated for a humid tropical climate to estimate grass reference evapotranspiration (ETo), using six methods for computing mean daily vapor pressure deficit (Δe). For each method, two f(u) were found, being one for the summer and another for the spring and fall seasons. Regardless of the method used, the slope of the lines obtained were parallel to the original f(u), indicating that the parameters **m** and **b** can be set equal to the original values, that is, **m** = 6.43 $MJ/m^2.d.kPa$, and **b** = 0.526 s/m. Thus, the only difference was found in parameter **a**, which is responsible for the vertical displacement of the regression line. For the summer, **a** varied from 0.41 to 1.0, while for the spring-fall seasons the range was -0.07 to 0.12. The seasonal change in the grass height cannot account for such distinction for, theoretically, the aerodynamics of the surface affects only the slope of the line. Relative ETo errors due to the original f(u) values were evaluated for each Δe method. Regardless of the method used, the errors were: (i) larger during the fall, in some cases in excess of 100%; (ii) between 10% and 50% during the spring; and (iii) around 10% for the summer.

Key words: Penman equation, reference evapotranspiration, daily mean vapor pressure deficit.

RESUMO

A função da velocidade do vento de Penman [f(u) = **m** (**a** + **b** u)] foi calibrada para um clima

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tropical úmido, para estimar a evapotranspiração de referência padrão grama (ETo), usando seis métodos para calcular a média diária do déficit de pressão de vapor d'água (Δe). Para cada método foram encontradas duas funções, sendo uma para o verão e outra para primavera-outono. Todos os métodos utilizados resultaram em retas com inclinação paralela à função original de Penman, indicando que os coeficientes **m** e **b** podem ser admitidos como iguais aos valores originais, isto é, **m** = 6,43 $MJ/m^2.d.kPa$, e **b** = 0.526 s/m. Portanto, a única diferênça foi encontrada no coeficiente **a**, que é responsável pelo deslocamento vertical da linha de regressão. Para o verão, **a** variou de 0,41 a 1,0, enquanto que para os períodos primavera-outono o intervalo foi -0,07 a 0,12. A variação sazonal na altura da grama não pode ser responsabilizada por tal deslocamento, pois a teoria mostra que a aerodinâmica da superfície afeta somente a inclinação da reta. Erros relativos na estimativa de ETo devidos aos parâmetros originais de Penman foram avaliados para todos métodos de Δe. Independentemente do método usado, os erros foram: (i) maiores durante o outono, sendo em alguns casos maiores que 100%; (ii) entre 10% e 50% durante a primavera; e, (iii) ao redor de 10% durante o verão.

Palavras-chave: evapotranspiração de referência, equação de Penman, deficit médio diário de pressão de vapor.

INTRODUCTION

PENMAN (1948) developed the combination equation for the computation of the daily rate of evaporation from open water (tank), bare soil and grass, based on the average meteorological data obtained at the weather station. Combining the sink strength and the energy balance, Penman proposed a combination equation which can be expressed as (DOORENBOS & PRUITT, 1977),

$$lE = W\ Rn + (1 - W)\ Ea,\ [MJ/m^2.d] \qquad \mathbf{1}$$

where, λE is the rate of evaporation, Rn is the net radiation, Ea is the sink strength for the water vapor, and W is a weighing factor dependent on the air temperature. For a natural surface, Penman considered that over a period of several days, and frequently over a single day, the change in stored heat in the soil is negligible when compared with the other terms of the energy balance equation. The first term on the right hand side of eq. (1) represents the diabatic contribution to the evaporation process, while the

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second term is the adiabatic contribution. They are also frequently referred to as the radiative and the convective terms, respectively.

The sink strength, that is, the drying power of the air, is expressed as a product of a wind function [f(u)] and the vapor pressure deficit of the air (Δe) at the mean air temperature, or,

$$Ea = f(u)\ \Delta e, \qquad \textbf{2}$$

which is a Dalton type equation. The wind function originally reported by PENMAN (1948), was found to be a linear relation of the wind speed at 2 m above the surface, that is,

$$f(u) = \mathrm{m}\ (\mathrm{a} + \mathrm{b\ u}). \qquad \textbf{3}$$

Results obtained during several years at Rothamsted, England, indicate that **m** = 6.43 MJ/m^2.d.kPa, **a** = 1, and **b** = 0.526 s/m gave the best fit, being **u** in m/s and Δe in kPa.

Eq. (3) is a statistical function and, obviously, its parameters values depend on the method of fitting the data and even though Penman recommended the above values as the best fit for practical use, he showed that, for some of his own data, different values would fit better. This is a clear indication that one should be suspicious when using the original Penman values, mainly in an extremely different climate.

Some time later, PENMAN (1956) revised his equation and proposed that the parameter **a** = 0.5 would give a better description of lake evaporation, but PENMAN (1963) returned to the original value **a** = 1. PRUITT (1960) found that **a** = 1 gave better results in estimating λE of ladino clover. DOORENBOS & PRUITT (1977), in a world wide publication, recommend the use of **a** = 1 and **b** = 0.864, but PRUITT & DOORENBOS (1977) cautioned that this, or another single function, cannot lead to generally reliable estimates of the reference evapotranspiration under a wide range of conditions; experimentally determined expressions for f(u) varied widely depending upon site and the method used to calculate Δe.

Penman's wind function has been extensively used throughout the world even though its parameters were determined in a temperate climate, and the objective of this paper is to test its goodness in a humid tropical climate.

THEORY

In essence, eq.(3) represents a bulk transfer coefficient for the water vapor without specifying explicitly the aerodynamic characteristics of the underlying surface. Evidently, a bulk transfer coefficient is not expected to work properly for every evaporating condition and van BAVEL (1966), and THOM &

OLIVER (1977) have taken Penman's approach and incorporated the surface characteristics through the use of the logarithmic wind profile. Combining eq.(2) and (3) results in

$$Ea_p = 6.43(1 + 0.526\,u)\Delta e, \qquad \mathbf{4}$$

where the subscript **p** indicates the results reported by Penman (1948). This notation was used by THOM & OLIVER (1977) to differentiate the Rothamsted results from those obtained elsewhere. It can be shown through the eddy diffusion theory that where, ñ is the air density, cp is the specific heat capacity of the air, $\lambda$is the psychrometric constant, being

$$Ea_p = r\ cp\ \Delta e/g\ \ ra_p\ , \qquad \mathbf{5}$$

$$ra_p = \frac{\mathrm{Ln}\left(\frac{2 - d_p}{zo_p}\right)}{u\ \ k^2} \qquad 6$$

the air resistance to the vapor transfer from the evaporating surface up to the reference height (z = 2 m); d_p and zo_p are, respectively, the displacement height and the roughness length, at the Rothamsted site; and k is the von Karman constant.

Applying eq.(5) and (6) elsewhere results in Ea and ra, respectively. Using the above set of equations, THOM & OLIVER (1977) demonstrated that

$$Ea = \frac{Ea_p * ra_p}{ra}, \ \text{ or}$$

$$\mathrm{Ea} = 6.43(1 + 0.526\ \mathrm{u})\Delta\,\mathrm{e}\,\frac{[\mathrm{Ln}]^2}{[\mathrm{Ln}]^2}, \qquad 7$$

is a theoretically sound extrapolation of the Penman wind function for any kind of surface. However, in order to match the original Penman's parameters values, the roughness length had to assume an unrealistically small value for the Rothamsted grass field, that is, zo_p = 0.00137 m. Through an empirically derived relationship between zo and the canopy height, STIGTER (1980) concluded that zo_p = 0.0085 m is a more realistic value. Taken the above two zo_p values one is left with a set of two general wind functions, or

$$f(u) = j\,6.43 \frac{1+0.526u}{\left(\mathrm{Ln}\dfrac{z-d}{zo}\right)^2}, \qquad \mathbf{6}$$

where, j = 53, for Thom - Oliver's zo_p, and j = 30, for Stigter's value. For a reference crop such as grass, with height between 0.08 m and 0.15 m (DOORENBOS & PRUITT, 1977) eq.(8) reduces to

$$f(u) = n\,6.43(1+0.526\,u)\,, \qquad \mathbf{7}$$

where, n is either 1.2 or 2.12, depending on which value is used for j.

MATERIAL AND METHODS

PENMAN (1948) has found f(u) to be a linear function of the wind speed. PRUITT & DOORENBOS (1977) have shown that the shape of the wind function is dependent on the way Äe is determined. DOORENBOS & PRUITT (1977) show distinct curvilinear shapes for f(u) at four out of ten sites when Äe was derived from a saturated vapor pressure based on the average of Tmax and Tmin. They also reported a wide variation of f(u) at Davis, California, between winter and summer seasons. CUENCA & NICHOLSON (1982) has indicated the correct methods of computing Δe corresponding to the various wind functions.

In this paper, the following six methods for computing the mean daily Δe were used:

METHOD 1:

$$\Delta e_1 = < es > - < ea >\,, \qquad \mathbf{8}$$

where, <es> and <ea> are, respectively, saturation and actual vapor pressure; the notation <> means that the average was obtained using the standard meteorological observations taken at the 9, 15 and 21 hours local time, which corresponds to 12, 18 and 24 hours GMT, the official time for weather observations in Brazil.

METHOD 2:

$$\Delta e_2 = es(Tave) - es(Tdmin), \qquad \mathbf{9}$$

where, Tave is the average of maximum and minimum temperature, and Tdmin is the minimum dewpoint temperature, which was obtained with the minimum temperature and the maximum relative humidity.

METHOD 3:

$$\Delta e_3 = es(Tave) - es(Tdave), \qquad 12$$

being, Tdave the daily average dewpoint temperature. This method is equivalent or close to the approach used by PENMAN (1948) and recommended in the FAO-24 modified Penman (DOORENBOS & PRUITT, 1977).

METHOD 4:

$$\Delta e_4 = es(Tave) * (1 - 0.01 * RHave), \qquad 13$$

where, RHave is the daily average relative humidity obtained from the thermohygrograph.

METHOD 5:

$$\Delta e_5 = 0.5[es(Tmax) + es(Tmin)] - es(Tdave), \qquad 14$$

where, Tmax and Tmin are, respectively, the maximum and minimum daily temperature. This is the method recommended by SMITH (1991).

METHOD 6:

$$\Delta e_6 = 0.5\{[es(Tmax) - ea(Tmax)] + [es(Tmin) - ea(Tmin)]\} \qquad 15$$

Grass (*Paspalum notatum* L.) evapotranspiration was measured by a lysimeter 6.7 m long, 4.5 m wide, and 1.5 m deep (VILLA NOVA & REICHARDT, 1989), located at the sugarcane experimental station (now School of Agriculture, of the Universidade Federal de São Carlos) in Araras, SP, Brazil (22°18'S; 47°23'W; 617 m), which has a Cwa type climate in the Köppen classification. Grass was clipped whenever necessary to keep its height between 0.08 m and 0.15 m, as recommended by FAO-24, in order to have an evapotranspiration representative of the reference value. The surroundings were also grown with the same grass. Water from a storage tank was continuously supplied from below, keeping the water level 0.30 m below the soil surface inside de lysimeter. The water tank had a system which recorded continuously, like a rain gauge, the water used by the grass. Due to the large volume of soil inside the lysimeter, there is a lag between the time the evapotranspiration takes place and the time the water tank senses the need to feed the soil to its initial water content. This problem makes the system suitable for daily and longer time scale measurements. A rain shelter was used to avoid disturbance on the measuring system during rainy periods and rainy days were eliminated from the analysis. Daily grass evapotranspiration was obtained from the spring of 1985 to the fall of 1986. All the necessary data were collected at a nearby (about 50 m away) weather station. Eq.(1) and (2) were used to determine f(u) for each Äe method, and correlated with the average wind speed, that is,

$$f_i(u) = \frac{lE - W\ Rn}{(1 - W)\Delta e_i} \quad i = 1,...,6 \qquad 16$$

Net radiation (Rn) was obtained from the relative sunshine duration (n/N) through following the seasonal equations developed by OMETTO (1973):

SPRING - SUMMER :

$$Rn = 8.28\ n/N + 6.86 \quad [MJ/m^2.d]$$

FALL - WINTER:

$$Rn = 5.48 \ n/N + 2.76 \quad [MJ/m^2.d] \qquad \mathbf{11}$$

The above relationships were developed for a nearby weather station (Piracicaba, SP, 22°42'S; 47°38'W; 546 m) and are site specific, that is, they have only regional significance since the coefficients where obtained through statistical regression between daily Rn, as measured by a Gier-Dunkle net exchange radiometer, and **n/N** being **n** taken from a Campbell - Stokes heliograph.

RESULTS AND DISCUSSION

Before beginning the discussion of the results obtained, it should be emphasized that the present work can be criticized for using Rn estimated through heliograph data (OMETTO, 1973) instead of the classical Brooks equation, as Penman did. However, it should be pointed out that the Brooks equation uses the mean daily water vapor pressure to estimate the atmospheric long wave emissivity, and that such variable depends strongly on the method used for its computation (see above), and this fact would bring another variable (Rn) into play, increasing the number of possible combinations. Obviously, any estimate brings an uncertainty but, in the present situation, it is expected that the approach used did not affect significantly the results.

As pointed out by PRUITT & DOORENBOS (1977) and reinforced by CUENCA & NICHOLSON (1982), the wind function f(u) was found to be dependent on the method used for computing the daily average vapor pressure deficit (Δe). For all Δe methods used, Figure 1 shows that f(u) for the summer was larger than that obtained for the spring and fall, which could well be represented by the same function. The original Penman function described well only the summer function for method 3, which is equivalent to the one used by PENMAN (1948). Also for the summer, the original Penman parameters resulted in acceptable fit, with small overestimation, when methods 2 and 4 were used. For the other seasons and for the other methods, the original f(u) represented an overestimation. For the spring and fall the overestimation was fairly gross, regardless of the Δe method used.

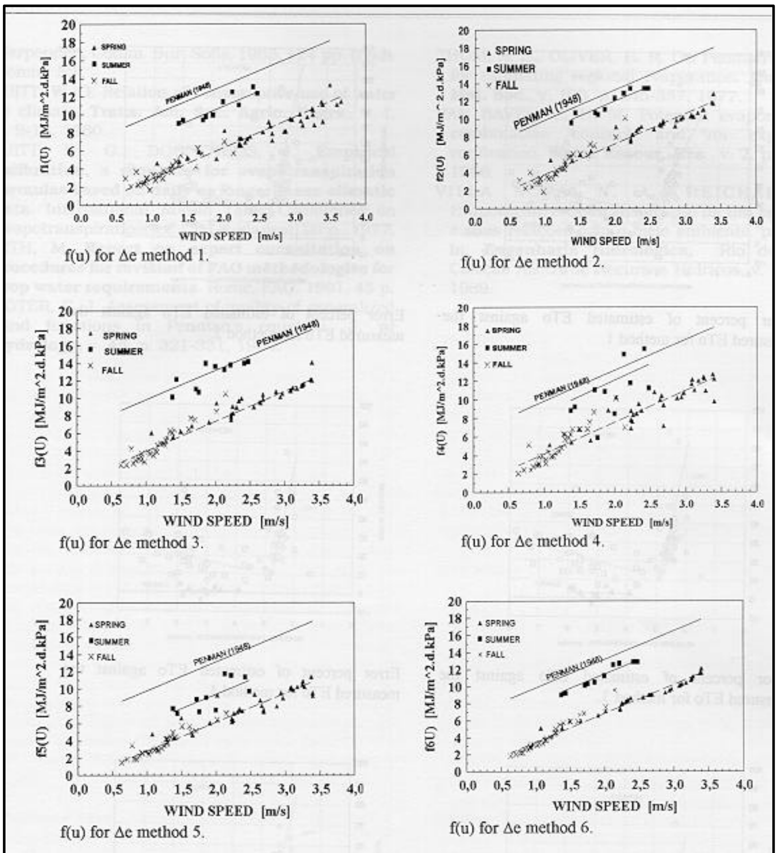


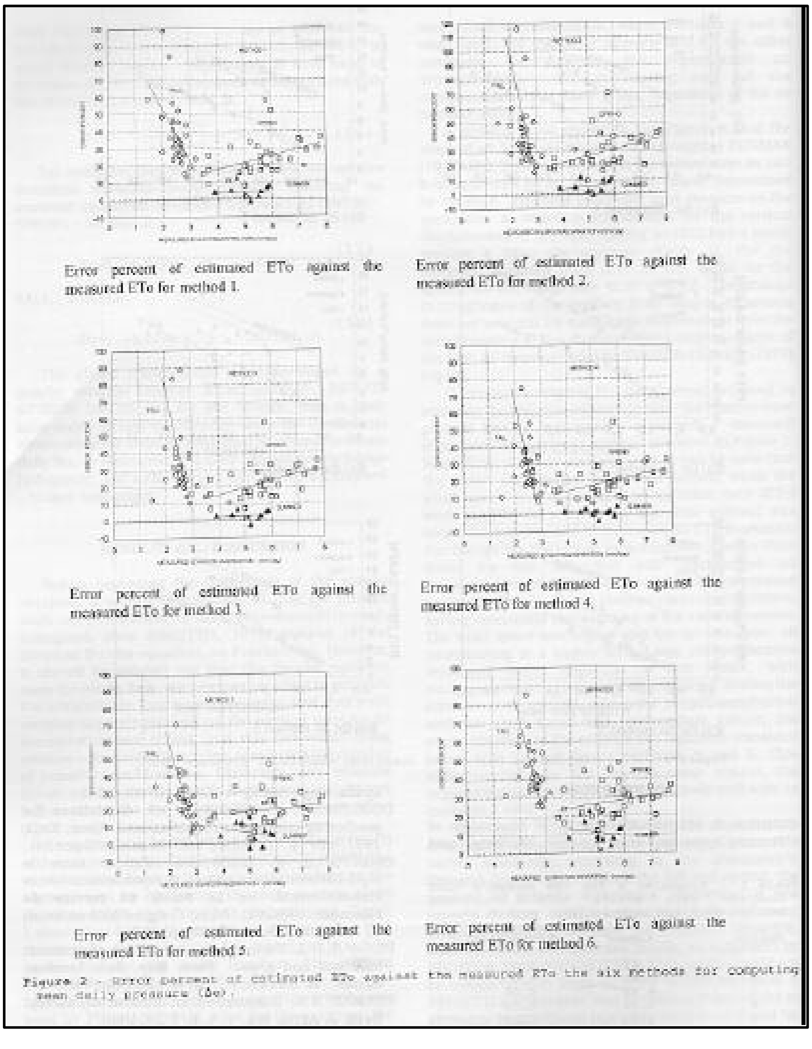
Figure 1 - f(u) for the six method for computing mean daily pressure (Δe).

Table 1 - Parameter **a** for the Penman's wind function with $m=6.43$ $MJ/m^2.d.kPa$, and $b=0.526$ s/m, for each Δe (kPa) method.

Method	Summer	Spring-fall
1	0.64	-0.07
2	0.82	0.07
3	1.00	0.09
4	0.83	0.12
5	0.41	-0.09
6	0.73	0.02

A striking point shown in the Figure is that the slope of all lines are parallel to the original PENMAN (1948) f(u). This indicates that the parameters **m** and **b** of eq.(3) can be set equal to the values determined by Penman. The only difference thus remains on the parameter **a**, which is responsible for the ertical displacement of the line. Spring and fall had a much smaller **a** than the summer (Table 1). For the summer, **a** varied from 0.41 to 1.0, while for the spring-fall the range was -0.07 to 0.12. The change in roughness of the surface from season to season does not account for such large difference in **a** for the aerodynamics of the surface affects only the slope of the line, as demonstrated by THOM & OLIVER (1977) [eq.(9)].

If Penman's original f(u) parameters are used to estimate the evapotranspiration rate, the relative error percent [100 * (estimated - observed) / observed] associated with each Δe method is shown in Figure 2. Regardless of the Δe method used, it can be seen that the errors were largest in the fall season, when the wind speed and the evapotranspiration rate (ETo) were smaller. In some days, the error percent was larger than 100%, and it decreased as ETo increased. For the spring, the errors were relatively smaller than those for the fall, but still represented an overestimation of 10% to 50%, and as ETo increased so did the errors. In the present climatic conditions, spring represents the begining of the rainfall season. The wind speed was higher and the air was drier, all contributing to a higher ETo. Large overpredictions represent faster depletion of soil water, with consequent mismanagement of irrigations. During the summer (wet season), when the evapotranspiration and the wind speed had intermediate values, the overestimation of the original f(u) equation remained below 10%, except for Δe methods 5 and 6. This indicates that, for the tropical rainy season, the original f(u) parameters worked relatively well with Δe methods 1 to 4.



Error percent of estimated ETo against the measured ETo for method 1.

Error percent of estimated ETo against the measured ETo for method 2.

Error percent of estimated ETo against the measured ETo for method 3.

Error percent of estimated ETo against the measured ETo for method 4.

Error percent of estimated ETo against the measured ETo for method 5.

Error percent of estimated ETo against the measured ETo for method 6.

Figure 2 - Error percent of estimated ETo against the measured ETo the six methods for computing mean daily pressure (Δe).

The results indicate that the relative contribution of the adiabatic term to the evapotranspiration rate varied seasonaly, according to the atmospheric demand, being higher during the fall and spring, the seasons of climatic transitions. Therefore, if one expects to have reliable estimates of the reference evapotranspiration through the Penman equation, then f(u) has to be calibrated locally, as suggested by PRUITT & DOORENBOS (1977). Our results could not be compared with those reported by DOORENBOS & PRUITT (1977) because they displayed their results in graphical form without reporting the values found for the f(u) parameters.

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